IOWA STATE UNIVERSITY
Aerospace Engineering Department

ENHANCED RANS MODELING OF SEPARATION-INDUCED TRANSITIONAL FLOWS USING FIELD INVERSION

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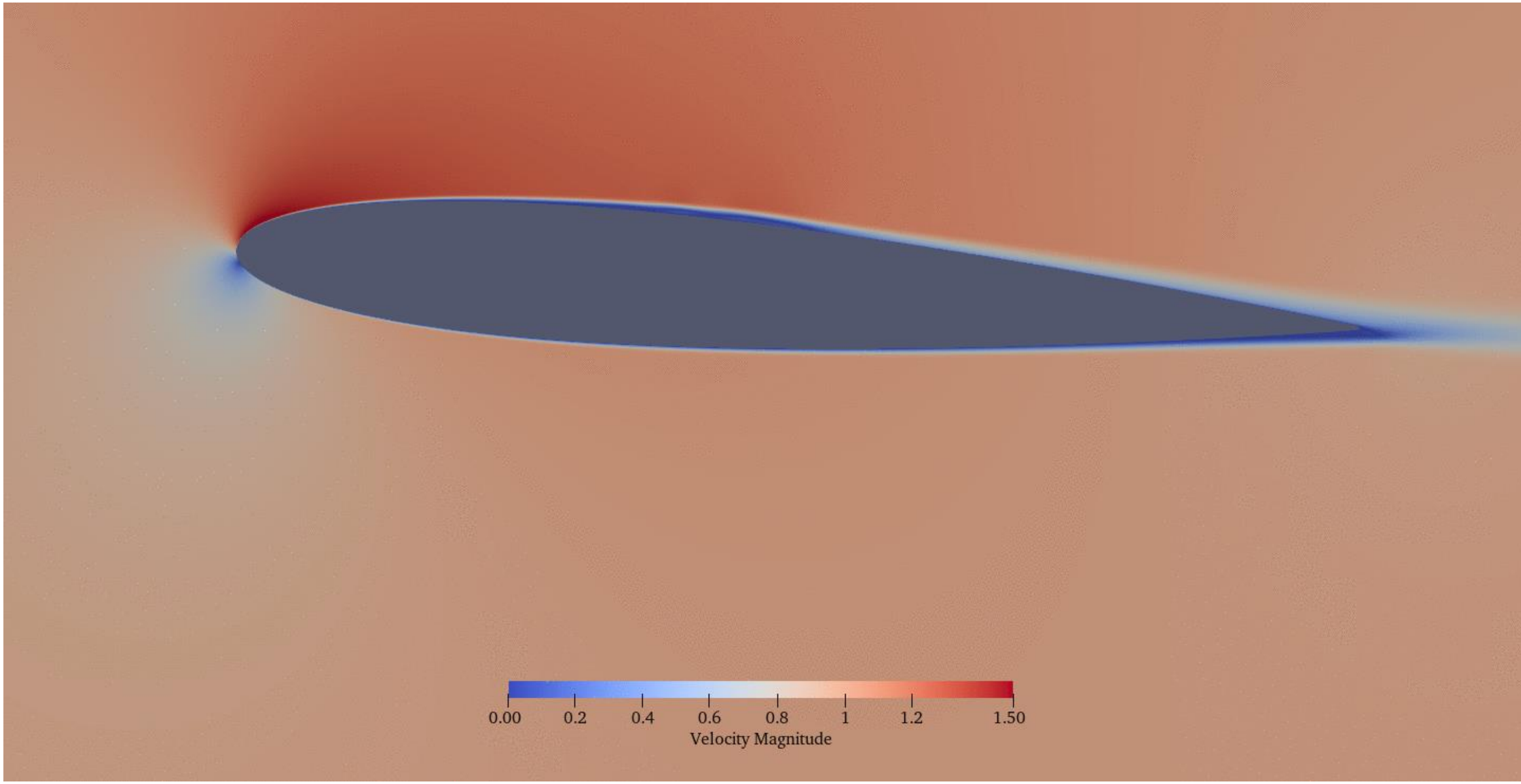
Dynamic Stall

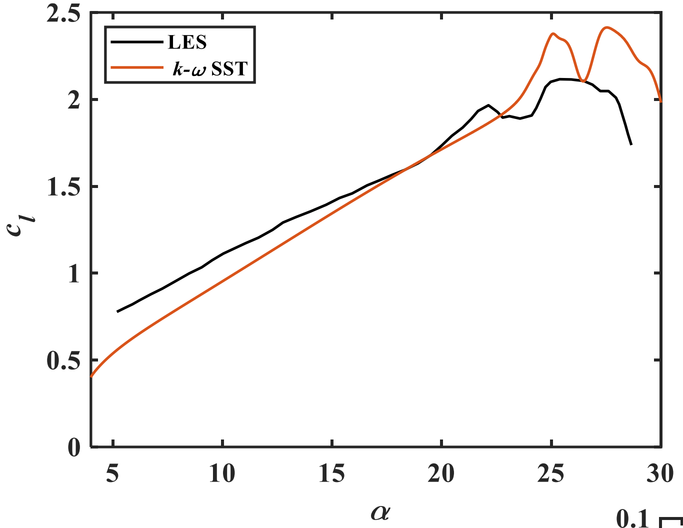
- Boundary layer acceleration (Magnus effect) causes delayed stall
- Laminar separation bubble bursting
- Formation of dynamic stall vortex and propagation
- This involves highly non-linear behavior that requires numerical investigations
- Existing RANS models need to be improved to better characterize dynamic stall

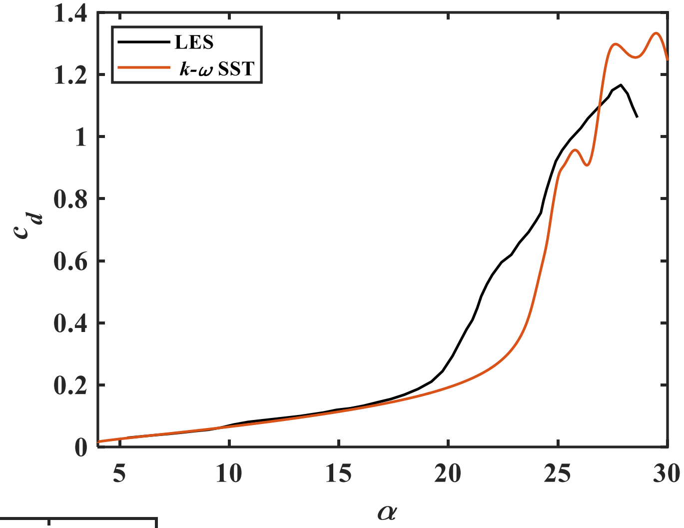
Dynamic Stall



NACA 0012 $Re = 200{,}000$ $\Omega = 2.86°/s$

Dynamic Stall

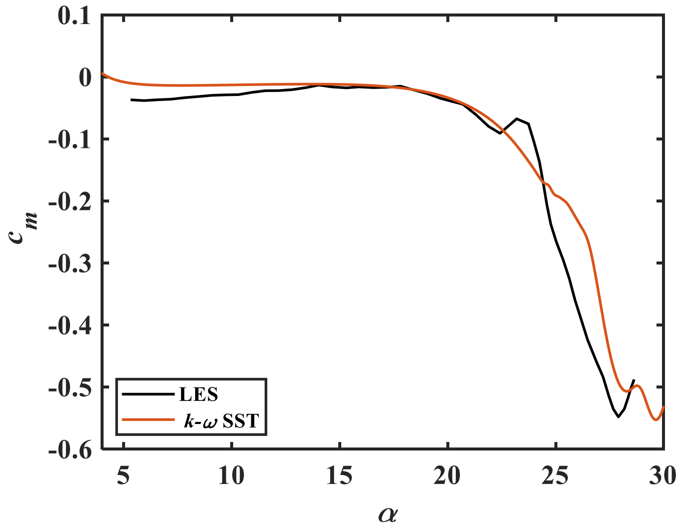




NACA 0015

$Re = 200{,}000$

$$\Omega_0^+ = \frac{\Omega_0 c}{U} = -0.05$$



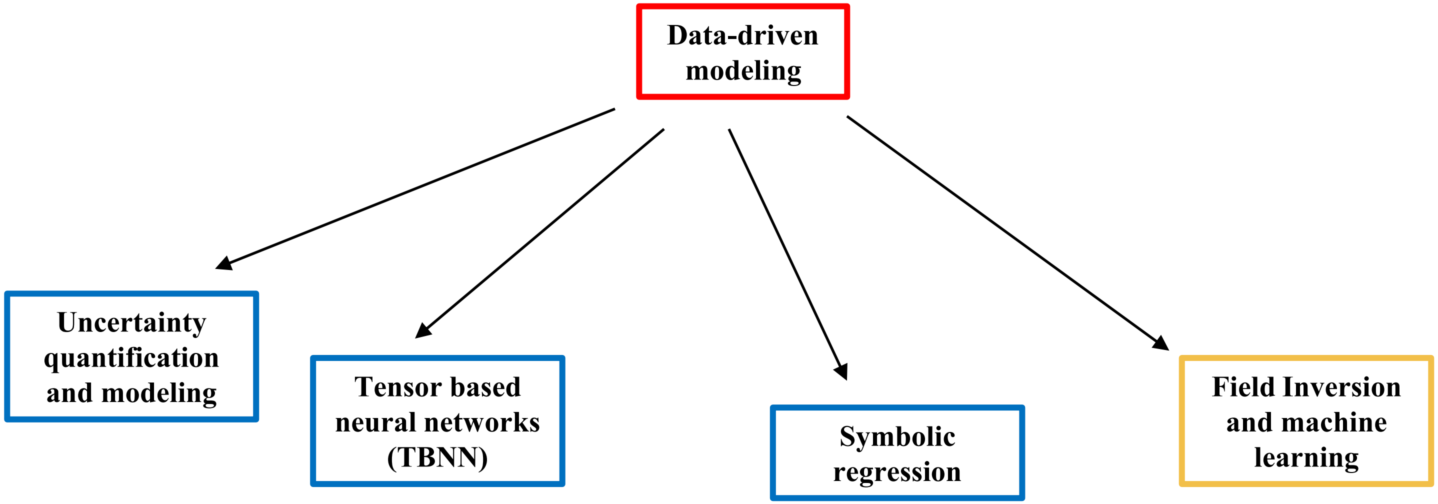
LES data is reproduced from: "Sharma and Visbal, Numerical investigation of the effect of airfoil thickness on onset of dynamic stall, JFM, 2019"

Research Objective

- Develop data-driven turbulence and transition models using steady-state data sets
- Evaluate the trained models on steady aerodynamic cases
- *Test the trained models on the pitching airfoil problem*

Data-Driven Modeling

- Scale resolving simulations (e.g. DNS and LES) are computationally expensive due to mesh and time resolution requirements
- Low fidelity cost-effective turbulence models (e.g. RANS) lack accuracy in non-equilibrium boundary layer flows and separated flows
- Laminar-to-turbulent transition even adds more uncertainty



Field Inversion and Machine Learning (FIML)

- Initially proposed by Karthik Duraisamy in 2014*
- Can work with limited data and is consistent with the predictive context
- Consists of three main steps:

1. Insert a corrective (discrepancy) field in the turbulence model

$$\frac{\partial \bar{\rho}\omega}{\partial t} + \frac{\partial \overline{U_j}\omega}{\partial x_j} = \beta(x) C_{\omega 1} \frac{\omega}{k} P - C_{\omega 2} \bar{\rho}\omega^2 + \frac{\partial}{\partial x_j}\left[(\mu + \sigma_\omega \mu_T)\frac{\partial \omega}{\partial x_j}\right]$$

2. Solve the inverse problem (optimization) to find β(x) that minimizes the discrepancy between the model and high-fidelity data (field inversion)

3. *Use a machine learning algorithm to train $\beta(x)$ against flow features*

K. Duraisamy and P. Durbin "Transition modeling using data-driven approaches, Center for Turbulence Research Proceedings of the Summer Program, 2014"

Data-Driven Transition Model

- Transition occurs through different mechanisms (natural, by-pass, separation induced, ..etc)
- FIML has been applied extensively to enhance RANS models for turbulent flows
- Less application to transition flows specially separation induced transition
- Two methods are proposed here:

1. Inferring the discrepancy field in a transition transport model
2. Inferring the intermittency field in a turbulence model (algebraic transition model)

Data-Driven Laminar Kinetic Energy Model (LKE)

- The baseline model follows the implementation Pacciani *et al.* (2011)[1] which has been previously used for data-driven modeling using symbolic regression[2]

$$\frac{\partial k}{\partial t} + U_j \frac{\partial k}{\partial x_j} = F_\mu P_k - C_\mu k\omega + R + \frac{\partial}{\partial x_j}[\left(\nu + \frac{\nu_T}{\sigma_k}\right)\frac{\partial k}{\partial x_j}]$$

$$P_k = \min(2\nu_T \frac{\partial U_i}{\partial x_j} S_{ij}, \frac{kS}{\sqrt{6}})$$ (Production limiter)

$$F_{mu} = \frac{\frac{1}{40}+\frac{Re_T}{6}}{1+\frac{Re_T}{6}} \qquad Re_T = \frac{\nu_T}{\nu}$$ (Damping function)

$$\frac{\partial \omega}{\partial t} + U_j \frac{\partial \omega}{\partial x_j} = 2C_{\omega_1} F_\mu |S|^2 - C_{\omega_2}\omega^2 + \frac{\partial}{\partial x_j}[\left(\nu + \frac{\nu_T}{\sigma_\omega}\right)\frac{\partial \omega}{\partial x_j}]$$

$$\frac{\partial k_l}{\partial t} + U_j \frac{\partial k_l}{\partial x_j} = \beta P_l - \varepsilon_l - R + \frac{\partial}{\partial x_j}(\nu \frac{\partial k_l}{\partial x_j})$$

Transfer

$$R = C_2 f_2 \beta^* f_2 \omega k_l$$

1. R. Pacciani, M. Marconcini, A. Fadai-Ghotbi, S. Lardeau, and M. Leschziner "Calculation of High-Lift Cascades in Low Pressure Turbine Conditions Using a Three-Equation Model, 2011"
2. Y. Fang, Y. Zhao, H. Akolekar, A. Sandberg, and R. Marconcini A data-driven approach for generalizing the laminar kinetic energy model for separation and bypass transition in low- and high-pressure turbines", 2023

Data-Driven Algebraic Transition Model

$$\frac{\partial k}{\partial t} + U_j \frac{\partial k}{\partial x_j} = \beta P_k - C_\mu k\omega + \frac{\partial}{\partial x_j}[\left(\nu + \frac{\nu_T}{\sigma_k}\right)\frac{\partial k}{\partial x_j}]$$

$$\frac{\partial \omega}{\partial t} + U_j \frac{\partial \omega}{\partial x_j} = \frac{C_{\omega_1}}{\nu_t} P_k - C_{\omega_2}\omega^2 + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_T}{\sigma_\omega}\right)\frac{\partial \omega}{\partial x_j}\right] + \frac{2(1-F_1)\sigma_{\omega_2}}{\omega}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j}$$

$\beta = 0$ (Laminar) $\beta = 1$ (Turbulent)

- The underlying $k - \omega$ SST model needs some running length to produce turbulent kinetic at low turbulent intensities and separation induced transition
- Hence, β is allowed to increase beyond the value of 1

Discrete Adjoint Method

- Objective function:

$$\beta = argmin[\sum_{wall}\left(\tau_w^{RANS} - \tau_w^{data}\right)^2 - \sum_{flow}\lambda(\beta - 1)^2]$$

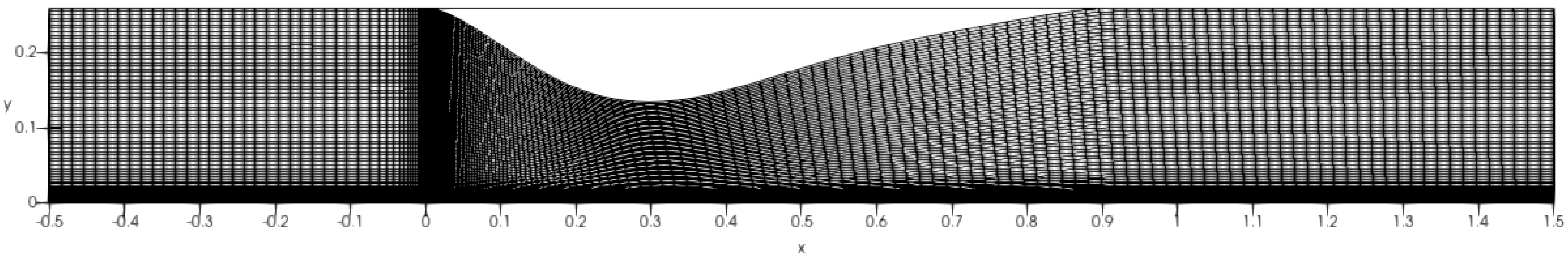
- DAFoam is used for field inversion
- An open-source code that inherits the OpenFOAM environment
- Equipped with the mechanics needed to:

1- Formulate the adjoint eqns.

2- Evaluate the partial derivatives using automatic differentiation

3- Solve the adjoint system of eqns.

Case 1: Flate Plate with Separation Induced Transition



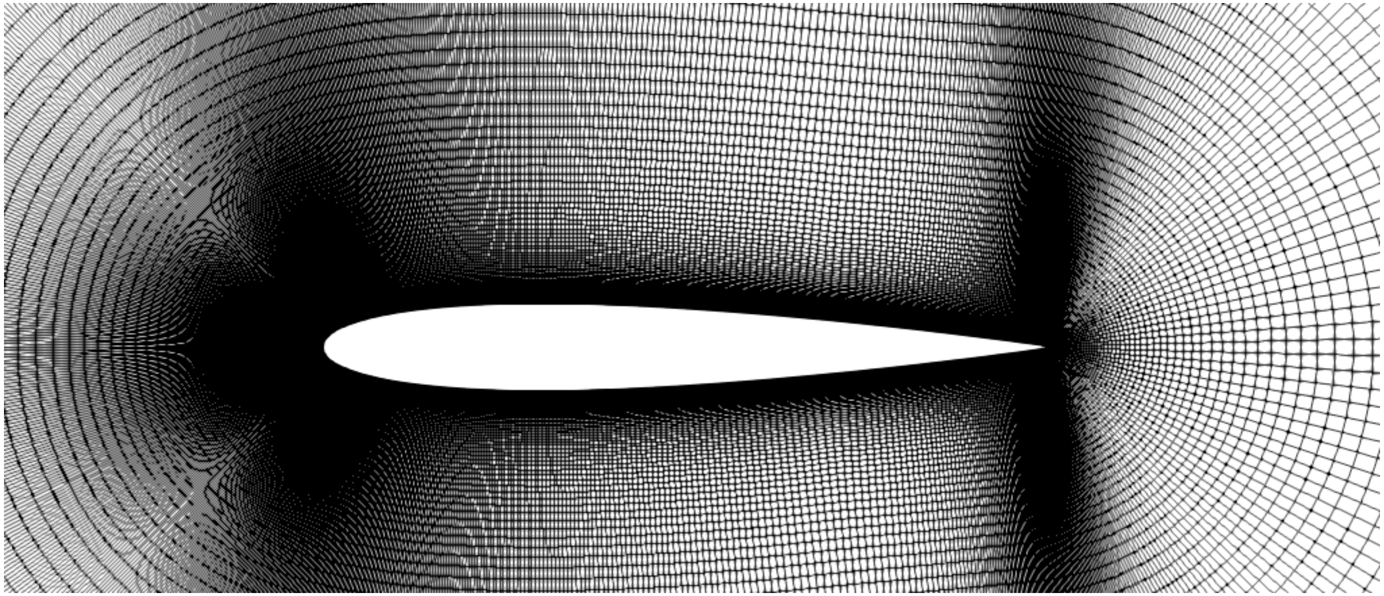
$U_{in} = 0.9$ m/s $\quad \nu = 1.5 \times 10^{-5}$

$Tu_1 = 5.8\%$ $\quad Tu_2 = 7.5\%$

Cells: $n_x = 149$ $\quad n_y = 99$

High-fidelity data: LES data by Lardeau et al. (2012)

Case 2: NACA 0012 Airfoil Series



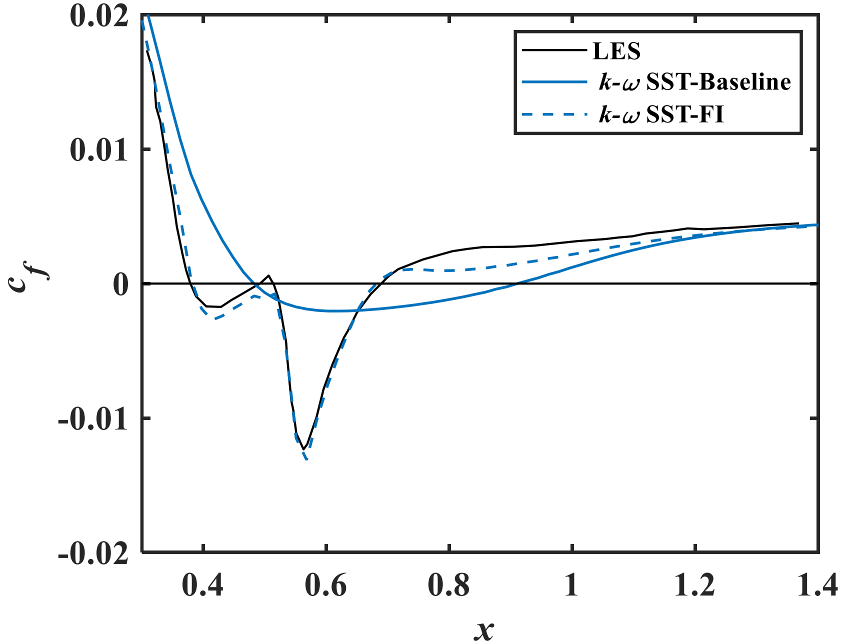
$U_{in} = 1$ m/s $\quad$ $Re = 200{,}000$

$Tu_{in} = 1\%$ $\quad$ $\alpha = 4°, 8°, 10°, 12°$

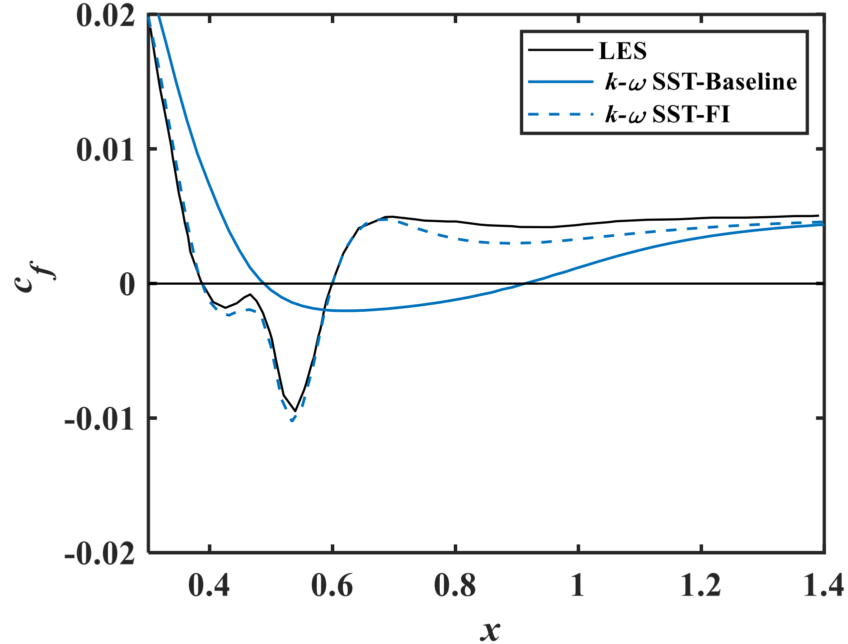
Cells: $n_{wall} = 887$ $\quad$ $n_{normal} = 180$

High-fidelity data: Conducted LES

Results: Flat Plate



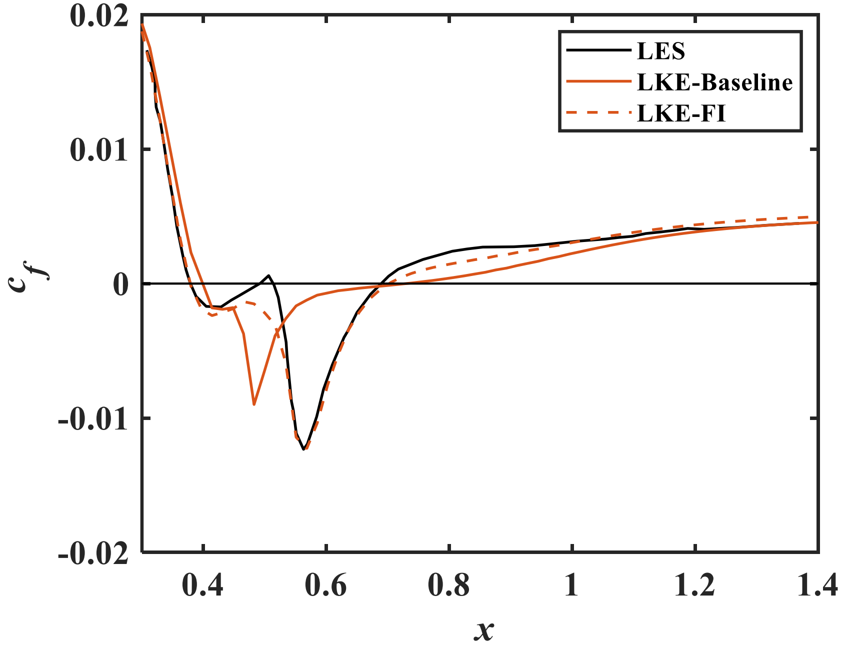
$Tu = 5.8\%$



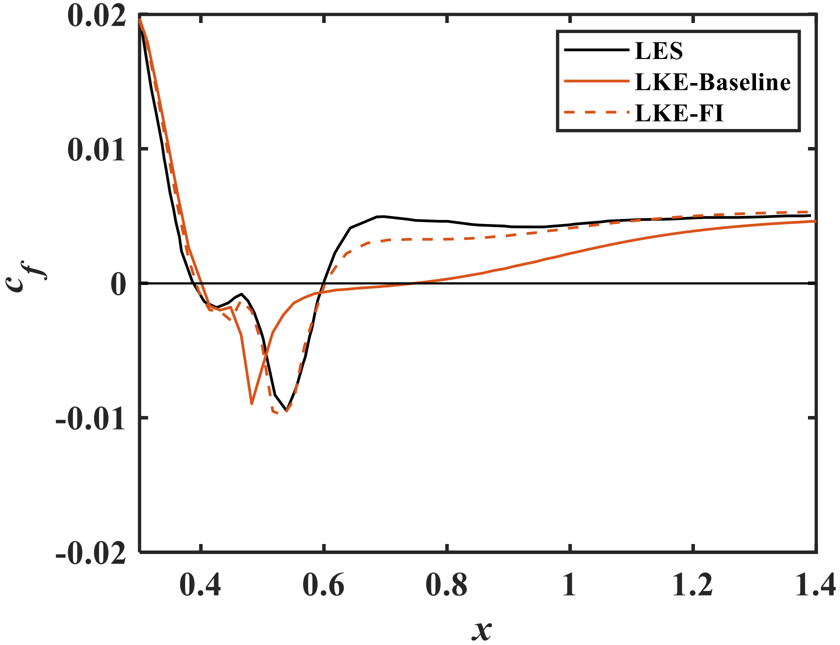
$Tu = 7.5\%$

Surface data is used up to just beyond the reattachment point

Results: Flat Plate



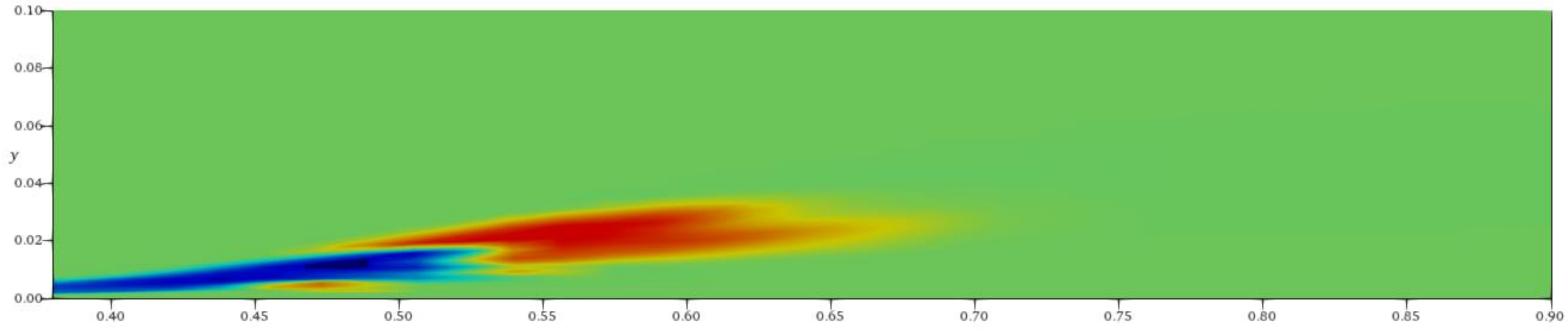
$Tu = 5.8\%$



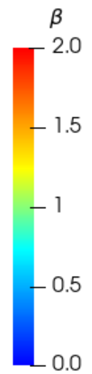
$Tu = 7.5\%$

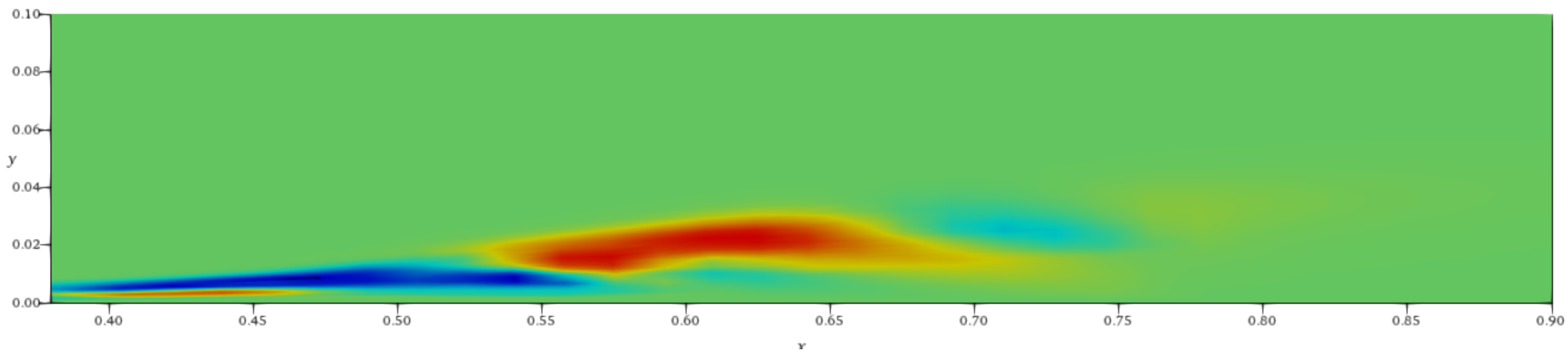
Entire surface data is used

Results: Flat Plate



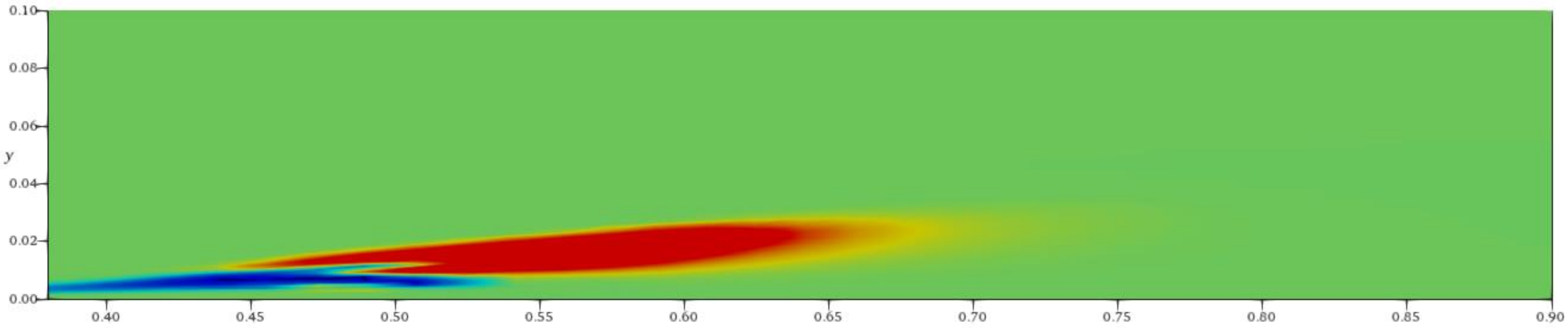
Field inversion for LKE at $Tu = 5.8\%$



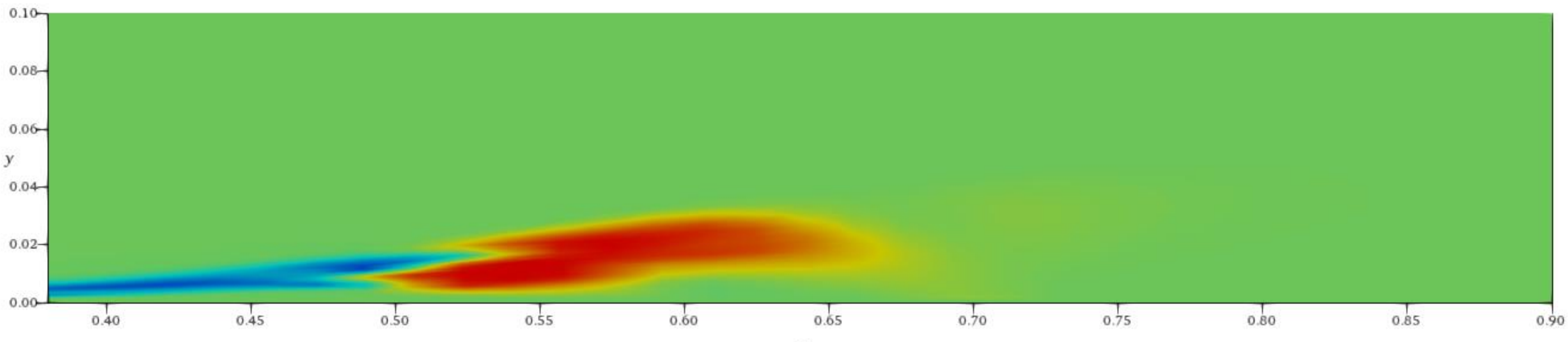


Field inversion for $k - \omega$ SST at $Tu = 5.8\%$

Results: Flat Plate

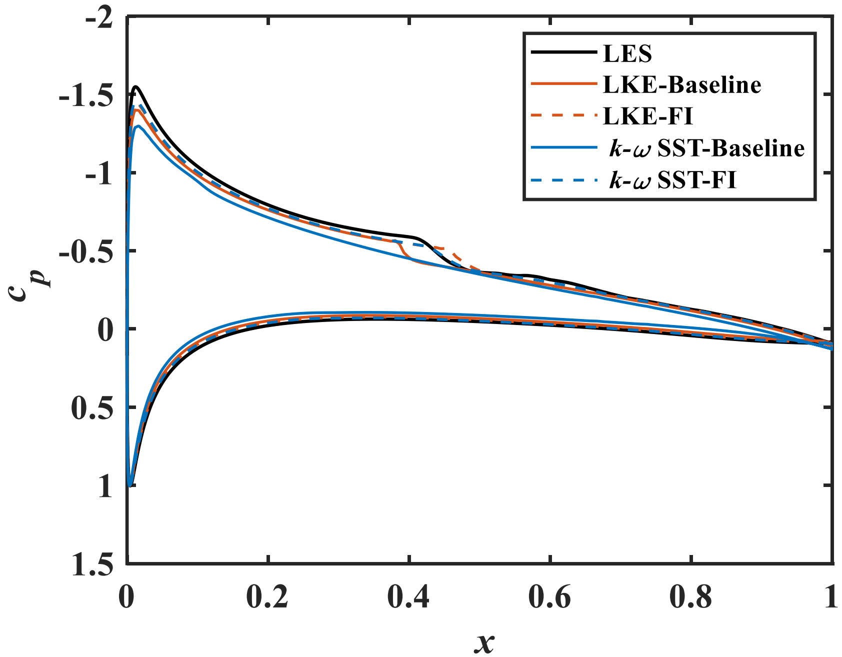


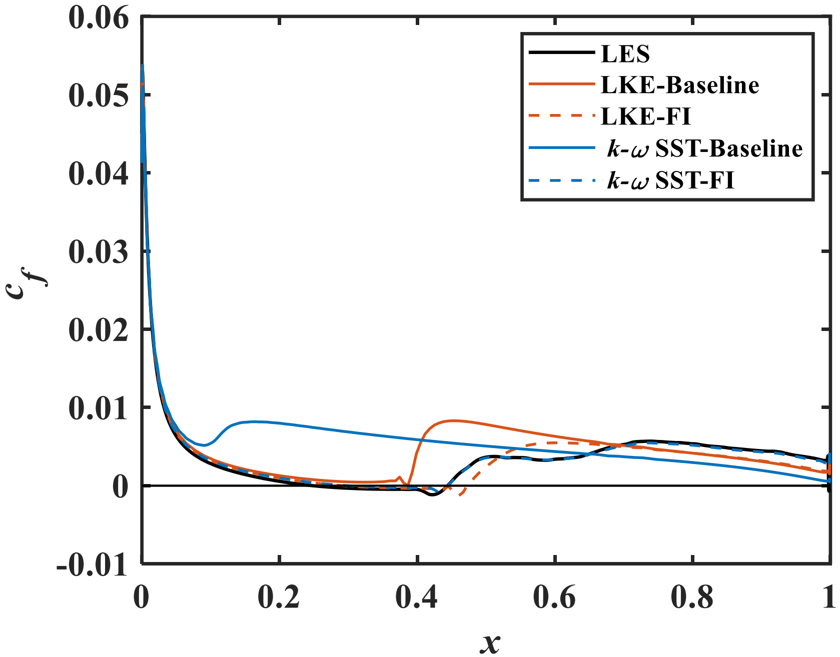
Field inversion for LKE at $Tu = 7.5\%$



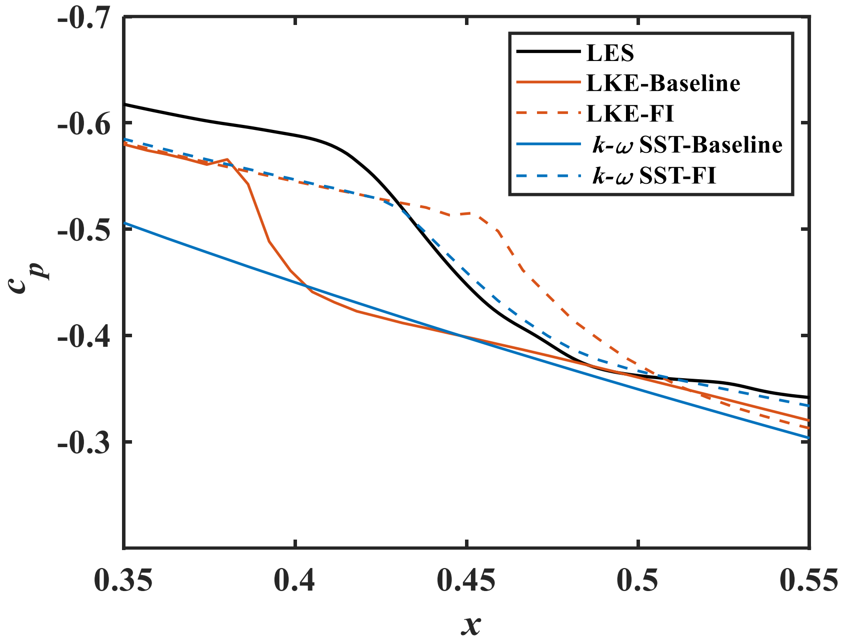
Field inversion for $k - \omega$ SST at $Tu = 7.5\%$

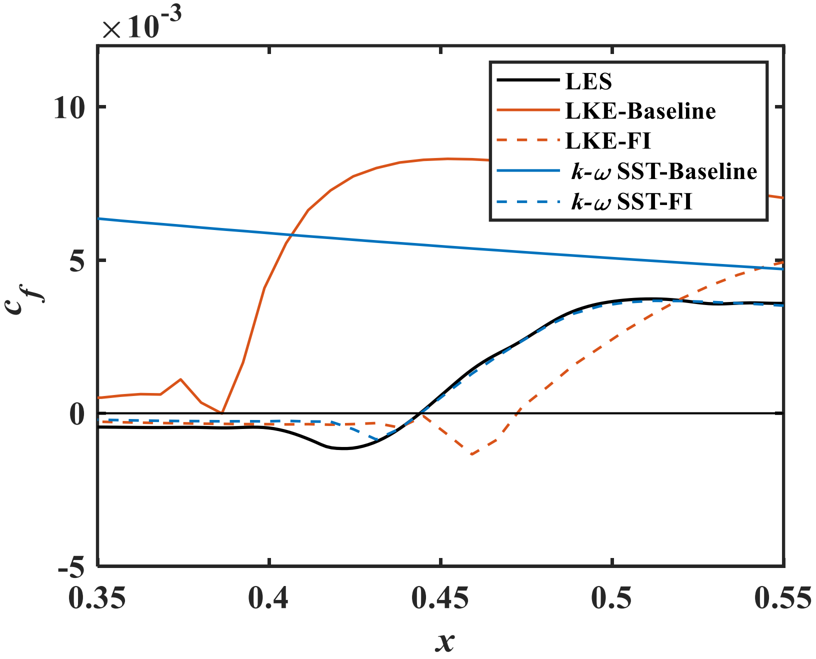
Results: NACA 0012-$\alpha = 4$



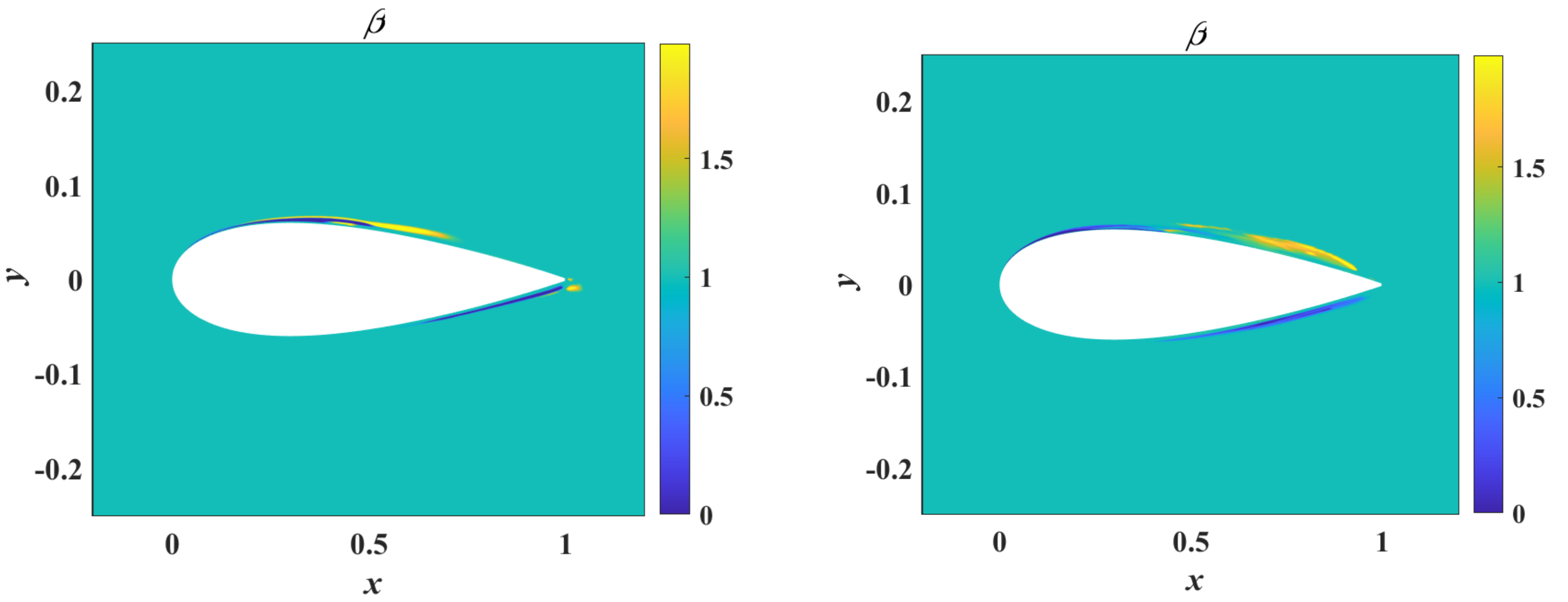


Results: NACA 0012-$\alpha = 4$

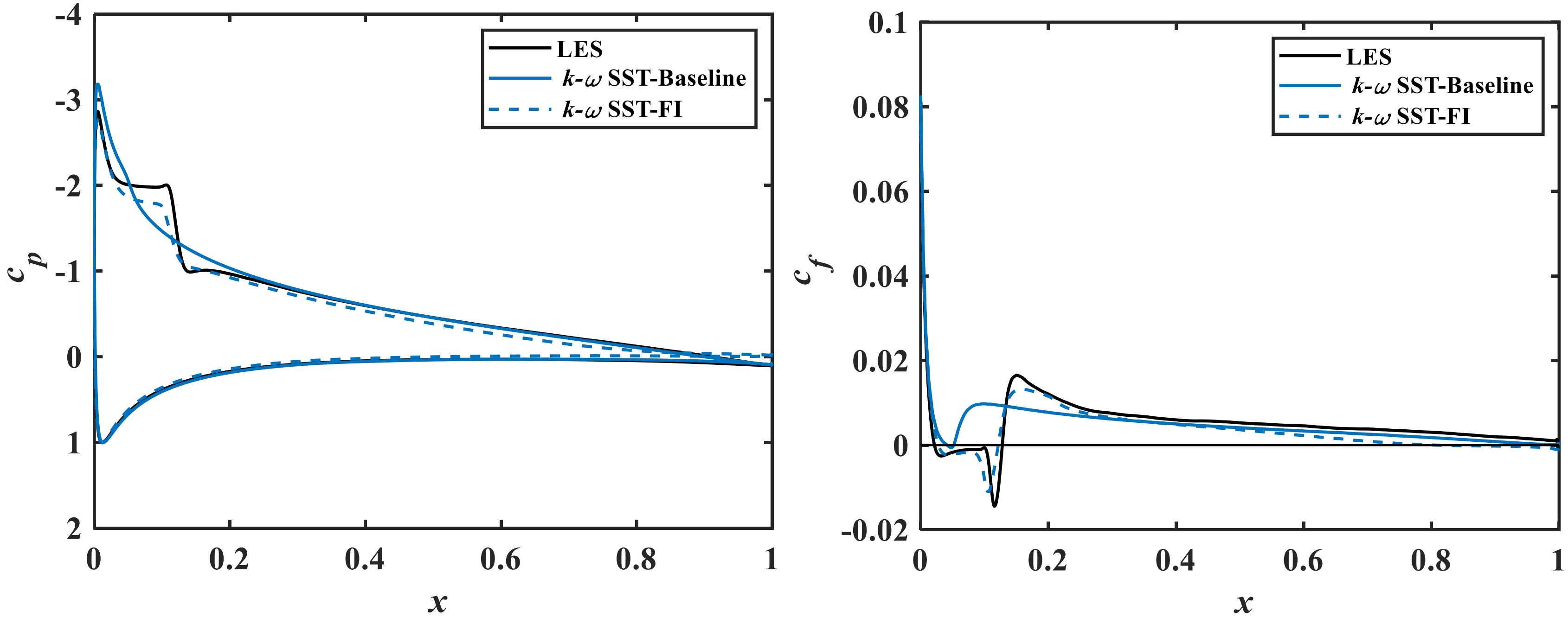




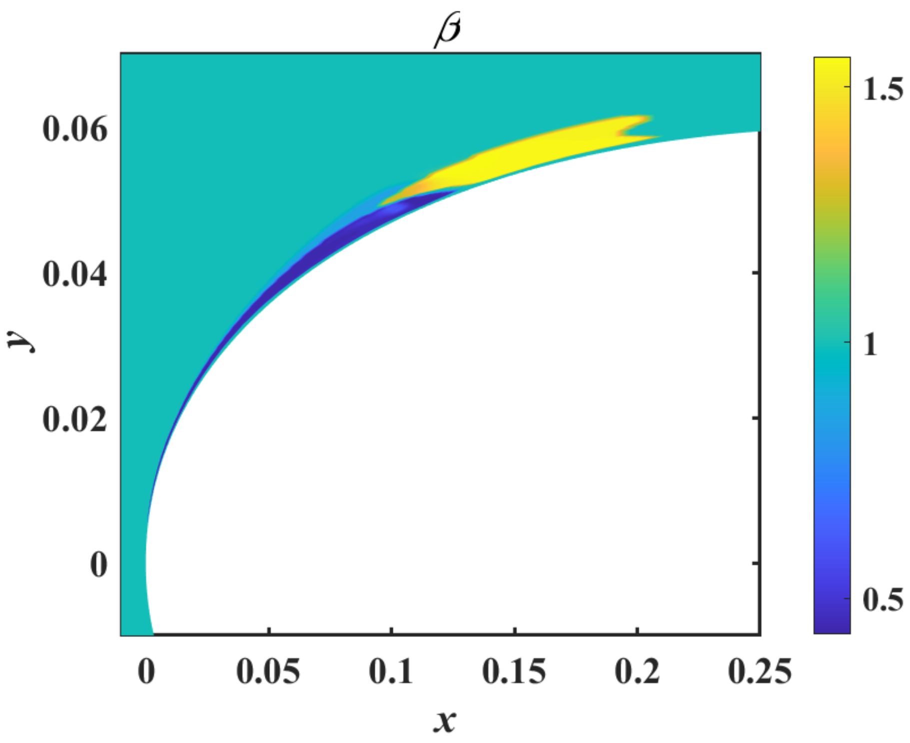
Results: NACA 0012-$\alpha = 4$



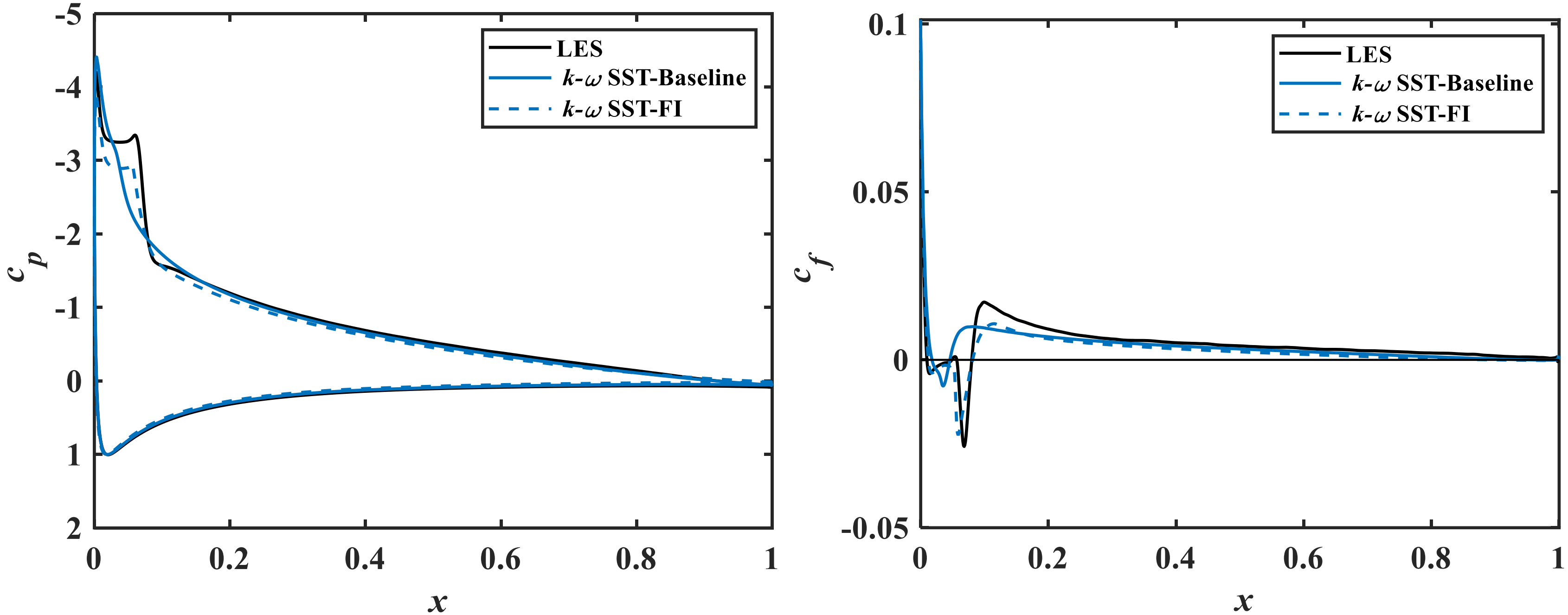
Results: NACA 0012-α =8



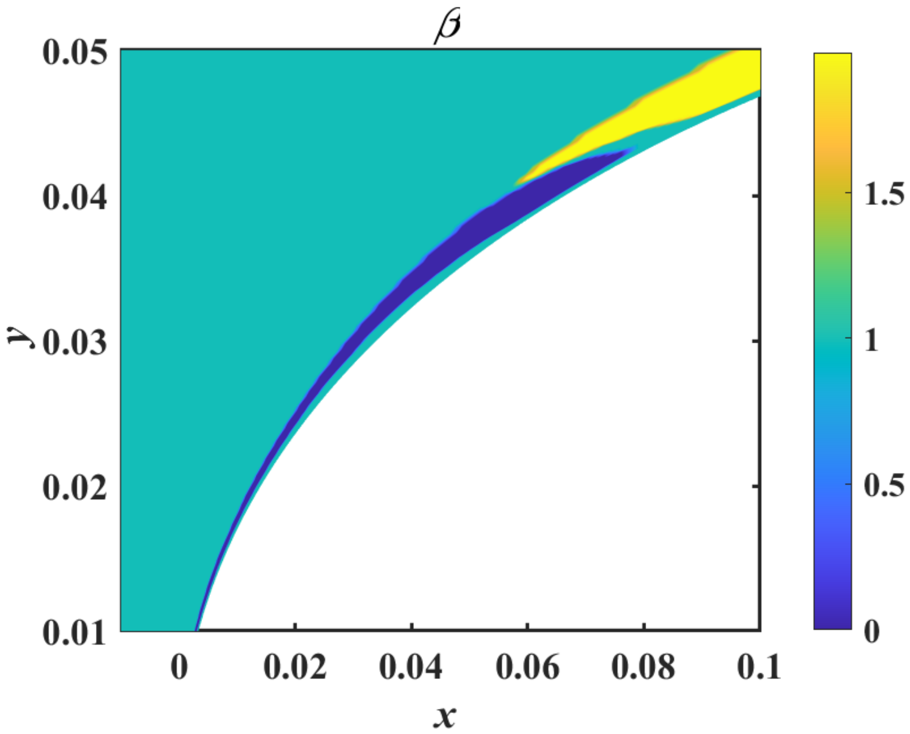
Results: NACA 0012-α =8



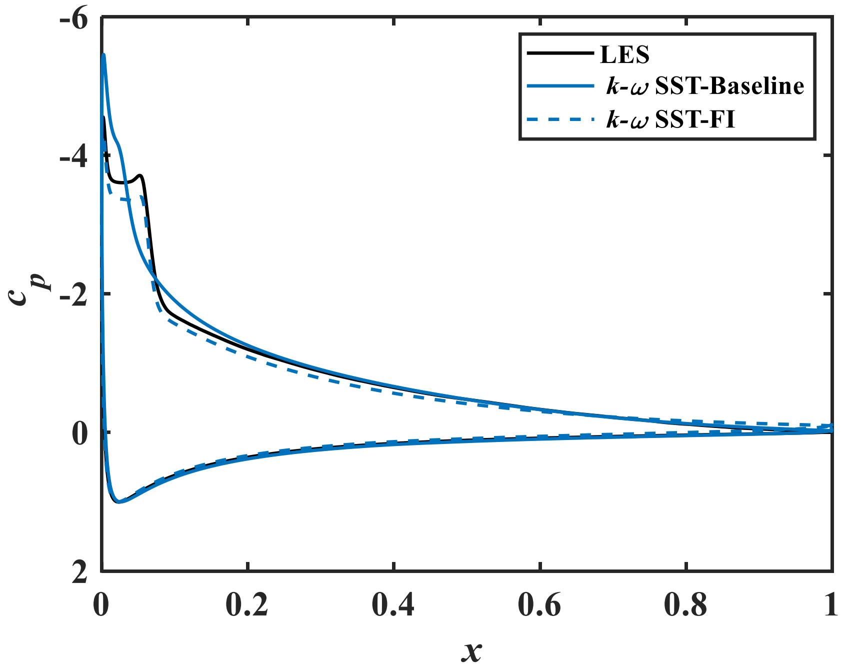
Results: NACA 0012-α =10

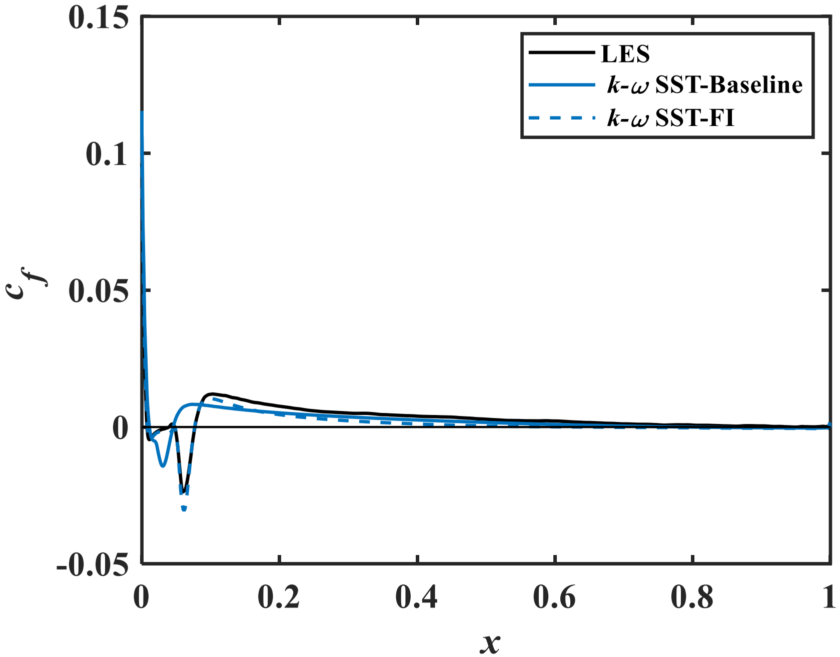


Results: NACA 0012-$\alpha = 10$

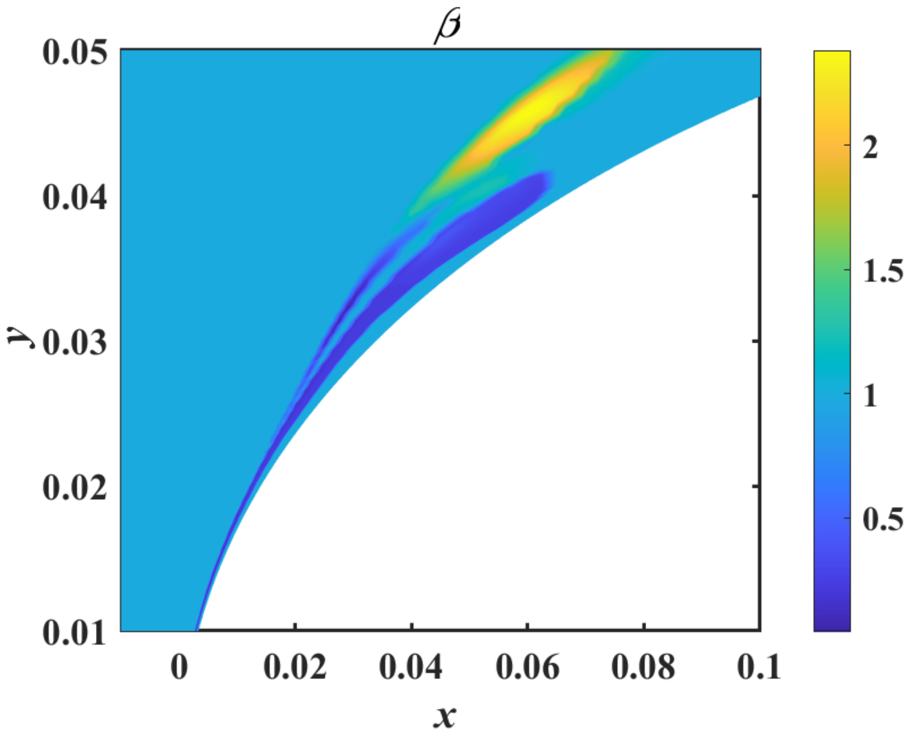


Results: NACA 0012-α =12





Results: NACA 0012-$\alpha = 12$



Conclusion and Future Work

- Field inversion has been shown to be effective for data-driven modeling
- Both models were able to fit with the high-fidelity data for the flat plate with FAPG case
- The algebraic model showed better convergence for the airfoil case
- *The field inversion data serves as input to train ML transition models*
- *Trained models will be applied to steady and unsteady aerodynamic problems*
- *Training models for fully turbulent flows will be considered*

TSFP13

- This work has been presented at the 13th international symposium on turbulence and shear flow phenomena (TSFP13) that was held in late June 2024 accompanied with a full paper.